

Model for Density Waves in Gravity-Driven Granular Flow in Narrow Pipes

Simen Å. Ellingsen

Department of Energy and Process Engineering, Norwegian University of Science and Technology, N-7491 Trondheim, Norway

Knut S. Gjerden, Morten Grøva, and Alex Hansen

Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway

(Dated: June 9, 2010)

A gravity-driven flow of grains through a narrow pipe in vacuum is studied by means of a one-dimensional model with two coefficients of restitution. Numerical simulations show clearly how density waves form when a strikingly simple criterion is fulfilled: that dissipation due to collisions between the grains and the walls of the pipe is greater per collision than that which stems from collisions between particles. Counterintuitively, the highest flow rate is observed when the number of grains per density wave grows large. We find strong indication that the number of grains per density wave always approaches a constant as the particle number tends to infinity, and that collapse to a single wave, which was often observed also in previous simulations, occurs because the number of grains is insufficient for multiple wave formation.

PACS numbers: 45.70.Mg, 83.10.Rs, 47.11.Mn, 47.57.Gc

I. INTRODUCTION

Transport of dry granular media through pipes and channels is a problem of fundamental interest which is of considerable industrial importance [1], and it has been studied intensely both experimentally and theoretically in recent decades [2–5]. Granular media behave radically different from both liquids and solids [3] and in granular pipe flow driven either by gravity or pressurized gas, non-linear dynamical phenomena such as clogging (density waves) are observed, but not yet well understood.

We consider dry grains falling inside a pipe. In this system, encountered industrially, e.g., in emptying of silos and transportation of sand or powder, there are three main mechanisms of interaction: (a) collisions between grains, (b) collisions between grains and walls, and (c) interaction of the grains with air in the system. Assuming the pipe is narrow, we approach this problem by means of a simple one-dimensional model with periodic boundary conditions in which collisions are modelled by means of two coefficients of restitution μ and ν , corresponding to the collisions of mechanisms (a) and (b), respectively. We demonstrate that this is sufficient to observe the formation of density waves.

Granular media has frequently been investigated by means of numerical simulations. Features such as clustering through dissipative collisions [6, 7] and inelastic collapse [8, 9] are among those reported. Various driving mechanisms have been considered, which can roughly be divided into two categories: vibrating walls [10–12] and interior heating [13–18]. The case of an emptying hopper has been studied, see e.g. [19, 20], as has the decompaction transient associated with the release of an initially dense packing [21].

The particular case of gravity-driven granular flow through channels and pipes has been studied for some time, both experimentally and theoretically. In previous experimental investigations [22–28] the importance

of the presence of air in the flow has been emphasized, and the formation of density waves in falling sand has been explained with primary reference to the air-grain interaction. Although clearly of importance in real systems, we demonstrate that the introduction of air in the model is not necessary to observe density waves. Our simulation suggests that such flocculent behaviour, while certainly influenced by the presence of air, can occur also in evacuated pipes, provided dissipation from collisions between grains and walls is faster than dissipation from grain-grain collisions.

Previous simulations of granular pipe flow have also reported density waves [6, 29–34]. Several mechanisms for velocity dissipation through inelastic collisions, damping and static friction have been employed, see, e.g., [35]. Early papers [6, 29] were unable to study sufficiently large numbers of grains and collisions to reach a state independent of initial conditions. The model used by Lee [30] was a 2+1-dimensional time-driven molecular dynamics (MD) simulation, with at least eight significant parameters. In the model of Peng and Herrmann [31, 32], a 2+1 dimensional lattice gas automaton is used in which various events are assigned probabilistic collision rules. Another model was proposed by Liss, Conway and Glasser [33]. Again, this is a 2+1 dimensional model with more complicated collision rules than ours. In their simulation the grain-wall coefficient of restitution was found to be of little importance. In all these models stable density waves were seen.

Our model is one-dimensional and much simpler, yet reproduces similar qualitative phenomena, dictated essentially by two coefficients of restitution whose interpretation is physically transparent.

We present our model and the results of our simulations and demonstrate that the model has two clearly distinct regimes in the parameter plane, one in which density waves form (flocculent regime) and one where no such are present (gaseous regime). Average number of

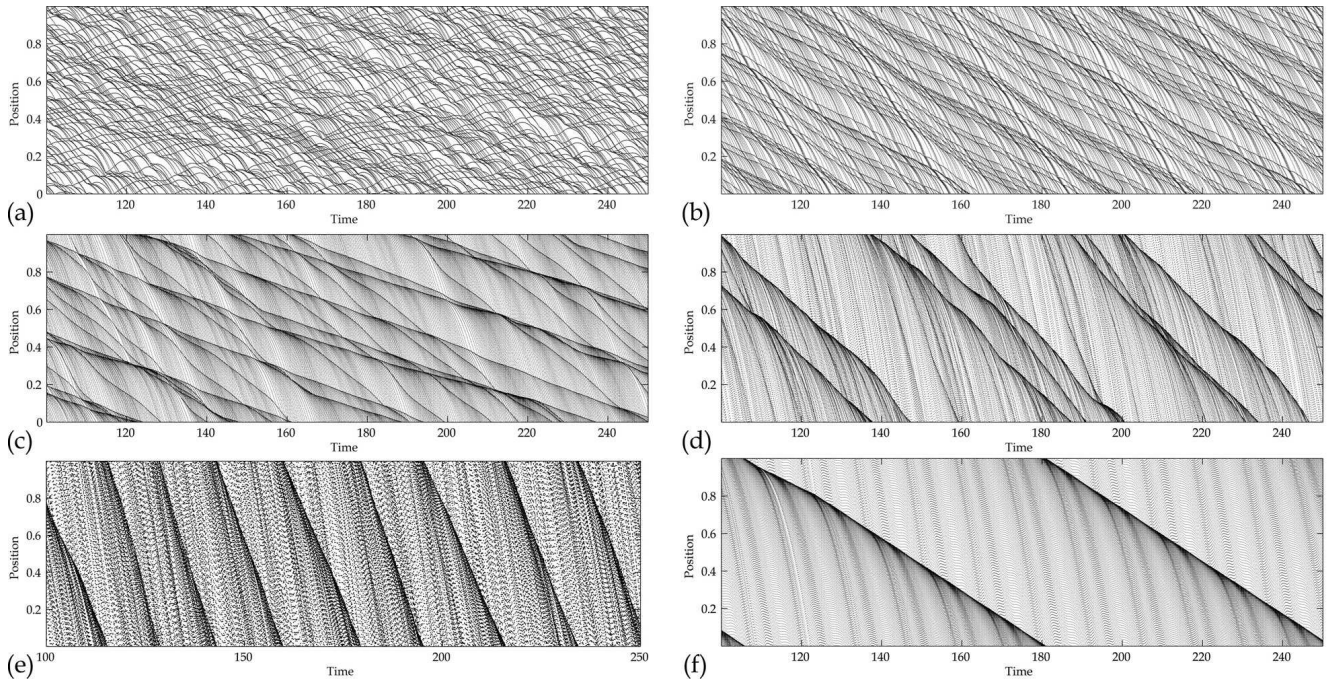


FIG. 1: Spatiotemporal plots of grain trajectories. (a) Gaseous regime $\mu = 0.8, \nu = 0.3$. (b) Split velocity transition point $\mu = \nu = 0.55$. (c) and (d): Different multi-wave patterns, $(\mu, \nu) = (0.55, 0.65)$ and $(0.30, 0.85)$, respectively. (e) and (f): Collapse to a single wave; $(\mu, \nu) = (0.70, 0.95)$ and $(0.15, 0.85)$, respectively. Further details in text.

grains per density wave and flow rate as functions of the two coefficients of restitution are studied.

II. THE MODEL

The pipe of our model is sufficiently narrow to prevent grains from passing each other, and sufficiently wide to allow them to fall approximately freely between collisions. As a consequence, grain-wall collisions are triggered by, and occur immediately subsequent to, grain-grain collisions. The pipe is therefore assumed to be too narrow for arching effects to play a role. We thus propose that the system is essentially one-dimensional in behavior and may be captured qualitatively by a one-dimensional model where collisions with walls are incorporated into the grain-grain collision rules themselves.

The set-up is the following. Let N grains move in one dimension so that grain i has position x_i and velocity v_i . We use periodic boundary conditions, with grains falling beneath $x = 0$ reinserted at $x = L$, where L is the length of the pipe. The grains are accelerated by a constant gravity g towards $x = 0$. In the dilute limit $dN \ll L$, where d is the diameter of a single grain, we find that the behaviour is independent of d , and we set $d = 0$ for simplicity.

As mentioned, the collision rules employed are based on the idea that the falling grains lose energy due to two different types of collisions, with the walls and with each other. Grain-grain collisions are modelled by reducing

the relative velocity of the colliding grains through a coefficient of restitution $\mu \in [0, 1]$. Thus, $v_{R,ab}^+ = -\mu v_{R,ab}^-$ where $v_{R,ab} = v_a - v_b$, grain ‘a’ is directly above grain ‘b’, and we use superscripts ‘-’ and ‘+’ to denote times just before and after a collision, respectively.

Grain-wall collisions tend to decrease the average velocity of the system, $v_{av} = N^{-1} \sum_{i=1}^N v_i$, which would otherwise diverge in time. Instead, v_{av} eventually reaches and fluctuates around a constant value at which dissipated energy is in equilibrium with energy gained from the gravitational field. We model the interaction with the walls at each collision by a coefficient of restitution $\nu \in [0, 1]$ which dissipates grain energy by reducing the center of mass (CM) velocity of the two colliding grains according to $v_{CM,ab}^+ = \nu v_{CM,ab}^-$, where the CM velocity of the pair is $v_{CM,ab} = \frac{1}{2}(v_a + v_b)$. These rules combine to

$$v_a^+ = \frac{1}{2}(\nu - \mu)v_a^- + \frac{1}{2}(\nu + \mu)v_b^- \quad (1a)$$

$$v_b^+ = \frac{1}{2}(\nu + \mu)v_a^- + \frac{1}{2}(\nu - \mu)v_b^- \quad (1b)$$

Note that introducing a restitution of v_{CM} , momentum is transferred for each collision from the grains to the (infinitely massive) pipe, and the momentum of the grains alone is not conserved. This causes the flow to approach a constant flow rate as observed in experiments.

While a collision changes the velocities of just two grains, the relative and CM velocities of three *pairs* of grains are affected: the colliding pair plus the neighboring pairs above and below. In particular, a collision increases the relative velocity of the pair above the collision

in proportion to the lost CM velocity of the colliding pair. Thus, relative velocities are constantly regenerated, and the phenomenon of inelastic collapse [8, 9] is avoided for $\mu > 0$ and $\nu < 1$.

III. SIMULATION AND RESULTS.

We use event-driven molecular dynamics for our simulations. Each grain is given an initial velocity which is assigned randomly according to various initialization schemes. The grains are also given equidistant starting positions between 0 and L .

Simulations are run for different grain numbers N at constant grain density $\rho = N/L$. After a ‘thermalization’ time the system reaches a steady state where the flow rate of the whole system fluctuates about a mean value. Initializing the simulations with different initial conditions yields the same dynamic steady state, characterized by average flow rate, collision frequency and the average number of density waves.

In Fig. 1 we plot grain trajectories in space and time. In these plots we have used, in arbitrary units, $L = 1, g = 0.01$ and $N = 100$. In these simulations initial velocities between 0 and 1 were drawn randomly from a uniform distribution.

We observe two distinct regimes in the μ, ν plane. When $\nu < \mu$, dense regions tend to spread out. In this ‘gaseous’ regime, shown in Fig. 1a, no steady density waves are observed. Whenever $\nu > \mu$, density waves are observed, and the transition from gaseous to flocculent behavior at $\mu = \nu$ is sudden.

For the special case $\mu = \nu$ seen in Fig. 1b, the eqs. (1) simplify to $v_a^+ = \mu v_b^-$ and $v_b^+ = \mu v_a^-$. Now the velocity of grain ‘a’ after a collision is only a function of the velocity of grain ‘b’ before the collision, and *vice versa*. After ‘thermalization’ we observe that the grain velocities are organized about two values: a lower velocity for the grains whose last collision was with the grain below it and a higher velocity for those which last collided with the grain above it.

While the qualitative behaviour in the gaseous regime, $\mu > \nu$, is insensitive to the values of μ and ν , the situation in the flocculent regime is quite different. As exemplified for the parameter pairs in Fig. 1, the wave patterns vary strongly in this half of the parameter plane. Both the magnitude and velocity of the density waves depend sensitively on the values of μ and ν . Moreover, the qualitative picture varies from a few large and stable waves with approximately constant velocity to many small and volatile waves which emerge, merge and dissolve and vary greatly in velocity even for a single set of parameters. Two examples are shown in Fig. 1c and d, but a multitude of different multiwave patterns may be observed.

Beyond rescaling the time axis, the values of L and g do not affect the wave patterns, so long as the grain number N is large compared to the average number of grains per

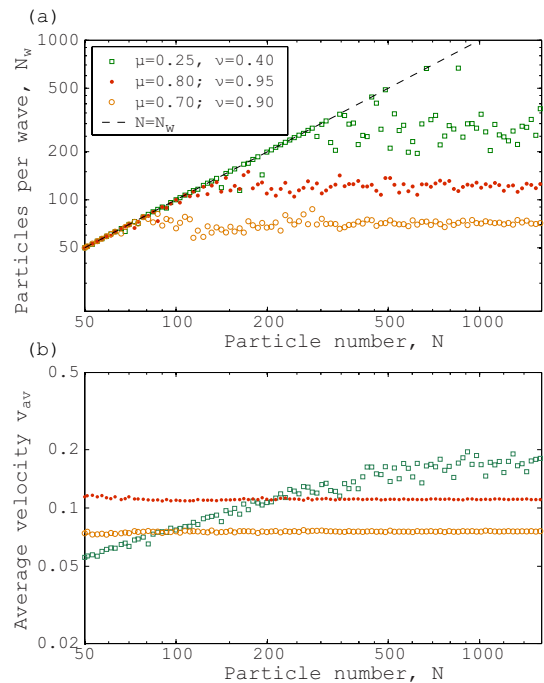


FIG. 2: (Color online) (a) Number of grains per waves and (b) average grain velocity as functions of particle number N for three sets of parameters (same in both panels). The dashed line in the above panel is $N = N_w$, i.e., a single density wave is present.

density wave. For N below some μ, ν -dependent threshold, the grains will gather in one or a few stable or metastable waves, as exemplified in panels (e) and (f) of Fig. 1.

When N is increased beyond this threshold, however, we find that the number of density waves increases linearly, while the number of grains per density wave remain constant. Average flow rate and the average velocity of density waves also remain independent of N in the limit of large N . This represents the asymptotic limit of our system, where there is no dependence on density ρ , which we illustrate for three pairs of coefficients in Fig. 2. Due to limitations of computer time we have not investigated this limit when N_w grows beyond ≈ 400 , which happens in the limit of either small μ , or $\nu \approx 1$, but we conjecture that such an asymptotic regime always exists when μ and ν are both on the open interval $(0, 1)$. Single-wave pictures such as panels (e) and (f) of Fig. 1, seen also in various other simulations [30–33], thus appear in our model simply because there are not enough particles in the system to form more than one wave [36].

In Fig. 3a we estimate the number of grains per wave in the flocculent regime, N_w , after ‘thermalization’ (calculated as N divided by the number of density waves) in the asymptotic (large N) regime. In practice we choose N large enough that the observed value of N_w is constant with increasing N . We have used N ranging from 500 to 4000 in different areas of the plotted region. N_w varies

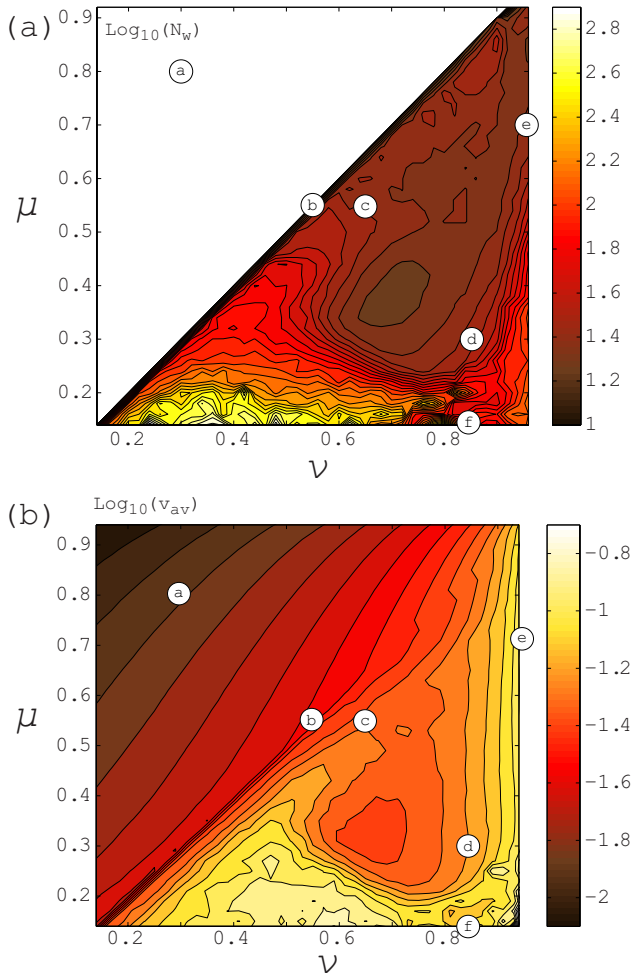


FIG. 3: (Color online) (a): Plot of $\log N_w$ where N_w is number of grains per density wave as a function of μ and ν in the asymptotic limit. (b): Plot of average flow rate as a function of μ and ν in the asymptotic limit. In both panels, encircled letters refer to parameter pairs in Fig. 1.

nonmonotonously and spans several orders of magnitude. Indeed, when μ decreases below 0.2 or ν approaches 1, both N_w and ‘thermalization’ time diverge rapidly, making calculation in this region expensive.

In Fig. 3b the average flow rate is given for all values of μ and ν . In the gaseous regime we see a monotonous variation supporting our observation that there is no significant parameter dependency on the structures formed in this region. Within the flocculent regime the pattern is similar to that observed for the number of grains per density wave.

Counterintuitively, flow rate is highest in areas of the μ, ν plane in which the density waves are large, and conversely, smaller waves correspond to a low flow rate. Since wave velocity is always lower than the flow rate, each grain will approach a density wave from above, collide its way through it, and fall into a low-density area beneath it once more before meeting another wave. Al-

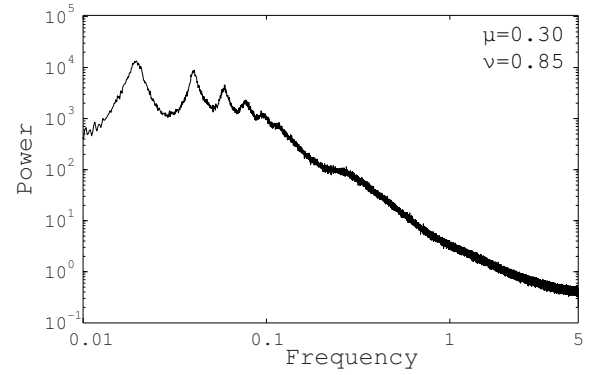


FIG. 4: Frequency spectrum of the wave pattern of Fig. 1d. The graph is the average over the power spectra of 200 sub-samples of a long time series of $\approx 10^7$ data points.

though large waves move more slowly than smaller waves, we find that this is more than compensated by the presence of large low density areas between waves in which the grains can fall freely. Hence the total flow rate is governed by the length of these acceleration stretches whereas we find no clear connection between flow rate and wave velocity.

We have analyzed the power spectrum of a typical wave pattern (that of Fig. 1d) shown in Fig. 4. The analysis was performed by recording density in a region of length $L/10$ as a function of time. With sampling frequency 10 per time unit, about 10^7 data points were recorded. The series was then divided into subsamples which were Fourier transformed individually, and the average of the power for all sub-samples were then taken (different number of sub-samples were used for comparison; 200 sub-samples are used in figure 4). The spectrum is peaked at a few frequencies, demonstrating how the wave pattern has a number of preferred wave-front velocities. This is not obvious from studying spatiotemporal diagrams such as Fig. 1d. In the high frequency regime the power dies off in a manner consistent with a power law with exponent -2, consistent with the wave-fronts undergoing Brownian fluctuations.

IV. CONCLUSIONS.

We have studied granular pipe flow by means of a one-dimensional two-parameter model. The two parameters are the coefficients of restitution for collisions between grains and collisions between grains and walls. The very simple collision rules are contrasted by the amount of structure exhibited by the model, and formation of density waves varying greatly in magnitude and qualitative behavior is observed. We find a criterion for the formation of density waves: that the dissipation from collisions with walls be greater than that from grain-grain collision. Under this criterion our model predicts that density waves can form also in the absence of any interstitial

gas.

Contrary to intuition, the flow rate is largest when density waves are large, slow and far between. This indicates that in some circumstances, the flow rate in gravity-driven granular pipe flow can be increased by softening or roughening the pipe walls. For example, with soft grains

described by $\mu = 0.3$, a 'rough' pipe with $\nu = 0.5$ gives a flow rate two to three times faster than a 'smoother' pipe for which $\nu = 0.7$.

We have benefited from discussions with J. P. Hulin and the Granular Media Group at FAST laboratory, Orsay.

-
- [1] R. Jackson, *The dynamics of fluidized particles* (Cambridge Univ. Press, 2000).
 - [2] M. A. Goodman and S. C. Cowin, *J. Fluid. Mech.* **45**, 321 (1971).
 - [3] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, *Rev. Mod. Phys.* **68**, 1259 (1996).
 - [4] H. Jaeger and S. Nagel, *Science* **255**, 1523 (1992).
 - [5] I. Goldhirsch, *Ann. Rev. Fluid Mech.* **35**, 267 (2003).
 - [6] M. A. Hopkins and M. Y. Louge, *Phys. Fluids A* **3**, 47 (1991).
 - [7] I. Goldhirsch and G. Zanetti, *Phys. Rev. Lett.* **70**, 1619 (1993).
 - [8] S. McNamara and W. R. Young, *Phys. Fluids* **4**, 496 (1992).
 - [9] S. McNamara and W. R. Young, *Phys. Rev. E* **50**, R28 (1994).
 - [10] S. Luding, E. Clément, A. Blumen, J. Rajchenbach, and J. Duran, *Phys. Rev. E* **49**, 1634 (1994).
 - [11] O. Herbst, P. Müller, M. Otto, and A. Zippelius, *Phys. Rev. E* **70**, 051313 (2004).
 - [12] O. Herbst, P. Müller, and A. Zippelius, *Phys. Rev. E* **72**, 041303 (2005).
 - [13] R. Caferio, S. Luding, and H. Jürgen Herrmann, *Phys. Rev. Lett.* **84**, 6014 (2000).
 - [14] D. R. M. Williams and F. C. MacKintosh, *Phys. Rev. E* **54**, R9 (1996).
 - [15] G. Peng and T. Ohta, *Phys. Rev. E* **58**, 4737 (1998).
 - [16] J. S. van Zon and F. C. MacKintosh, *Phys. Rev. Lett.* **93**, 038001 (2004).
 - [17] J. S. van Zon and F. C. MacKintosh, *Phys. Rev. E* **72**, 051301 (2005).
 - [18] D. J. Bray, M. R. Swift, and P. J. King, *Phys. Rev. E* **75**, 062301 (2007).
 - [19] G. W. Baxter, R. P. Behringer, T. Fagert, and G. A. Johnson, *Phys. Rev. Lett.* **62**, 2825 (1989).
 - [20] G. H. Risto and H. J. Herrmann, *Phys. Rev. E* **50**, R5 (1994).
 - [21] L. Conrath and J. M. Salazar, *Gran. Matter* **2**, 47 (2000).
 - [22] O. Moriyama, N. Kuroiwa, and M. Matsushita, *Phys. Rev. Lett.* **80**, 2833 (1998).
 - [23] G. Reydellet, F. Rioual, and E. Clément, *Europhys. Lett.* **51**, 27 (2000).
 - [24] Y. Bertho, F. Giorgiutti-Dauphiné, and J.-P. Hulin, *Phys. Fluids* **15**, 3358 (2003).
 - [25] J.-L. Aider, N. Sommer, T. Raafat, and J.-P. Hulin, *Phys. Rev. E* **59**, 778 (1999).
 - [26] S. Horikawa, T. Isoda, T. Nakayama, A. Nakahara, and M. Matsushita, *Physica A* **233**, 699 (1996).
 - [27] T. Raafat, J. P. Hulin, and H. J. Herrmann, *Phys. Rev. E* **53**, 4345 (1996).
 - [28] Y. Bertho, F. Giorgiutti-Dauphiné, T. Raafat, E. J. Hinch, H. J. Herrmann, and J. P. Hulin, *J. Fluid Mech.* **459**, 317 (2002).
 - [29] T. Pöschel, *J. Phys. I France* **4**, 499 (1994).
 - [30] J. Lee, *Phys. Rev. E* **49**, 281 (1994).
 - [31] G. Peng and H. J. Herrmann, *Phys. Rev. E* **49**, R1796 (1994).
 - [32] G. Peng and H. J. Herrmann, *Phys. Rev. E* **51**, 1745 (1995).
 - [33] E. D. Liss, S. L. Conway, and B. J. Glasser, *Phys. Fluids* **14**, 3309 (2002).
 - [34] C. Becco, H. Caps, S. Dorbolo, C. Bodson, and N. Vandewalle, in *Traffic and Granular Flow '03*, edited by S. P. Hoogendorn et al. (Springer, Berlin, 2005), pp. 543-551.
 - [35] J. Lee and H. J. Herrmann, *J. Phys. A* **26**, 373 (1993).
 - [36] Naturally we cannot conclude that the same is necessarily true for the simulations of Refs. [30–33].